Problem 1. (7 pts) Concussions can result from banging one’s head against a rigid surface (like, say, the floor of a skateboard halfpipe, or a brick wall, or the side of a lamppost). Why does a padded helmet reduce the likelihood of a concussion? Be mathematically precise.

Concussions are due to the sudden deceleration of your head that occurs upon striking an unmoving object.

- This deceleration can be expressed as:
  \[ |\vec{a}| = \frac{V_0 - 0}{\Delta T} \]
  \[ \Rightarrow \text{Time required to bring head to a stop after initial contact} \]
  \[ \Rightarrow \text{Magnitude of deceleration} \]

- A padded helmet increases the time needed to bring your head to a stop
  \[ \Rightarrow \Delta T \text{ increases} \]
  \[ \Rightarrow |\vec{a}|, \text{ deceleration magnitude decreases} \]
  So concussion likelihood decreases.
Problem 2. (7 pts) A very long train travels on a perfectly straight, horizontal track, without any friction, at a constant speed. Then rain starts to fall vertically from an enormous cloud. The tops of the train cars are open, so water can accumulate inside. Is it necessary to apply force to the train in order to maintain the same constant speed? If so, what’s the direction of the required force?

Consider a system that comprises the train cars and all of the rain. If there are no external forces in the direction of the train’s motion (note: gravity is \( \perp \) to this direction), then the system’s momentum in that direction is constant.

\[ P_{\text{system}} = M_{\text{train}} V_{\text{train}} + M_{\text{H}_2\text{O}, \text{train}} V_{\text{train}} + M_{\text{H}_2\text{O}, \text{rain}} \cdot 0 \]

This is the mass of water moving with the train.

\[ = (M_{\text{train}} + M_{\text{H}_2\text{O}, \text{train}}) V_{\text{train}} \text{ is constant if no external forces.} \]

(2 pts)

As time passes, \( M_{\text{H}_2\text{O}, \text{train}} \) increases, so \( V_{\text{train}} \) decreases if no external forces.

(2 pts)

You need a force in the direction of the train’s motion to maintain a constant \( V_{\text{train}} \) as \( M_{\text{H}_2\text{O}, \text{train}} \) increases.

(3 pts)
Problem 3. (12 pts) Suppose we can model an ice skater as three cylinders - the 'body' cylinder has length \( L_B = 1.62 \text{ m} \), diameter \( D_B = 0.23 \text{ m} \), and mass \( M_B = 44 \text{ kg} \), while the 'arm' cylinders each have length \( L_A = 0.71 \text{ m} \), diameter \( D_A = 0.10 \text{ m} \) and mass \( M_A = 3 \text{ kg} \). The mass in each cylinder is distributed uniformly. During her performance, the skater starts a spin, arms extended, with angular velocity \( \omega_1 = 8 \text{ radians/sec} \) around her body's long axis. What's her angular velocity, \( \omega_2 \), after she pulls her arms straight down to her sides? Assume that no friction acts on the skater.

\[
\begin{align*}
\text{I}_1 : \quad \text{I}_1 &= \text{I}_{\text{body}} + 2 \cdot \text{I}_{\text{arms}} \\
&= \frac{1}{2} M_B \left( \frac{D_B}{2} \right)^2 + 2 \cdot \frac{1}{12} M_A \left( \frac{L_A}{2} \right)^2 + \frac{1}{4} M_A \left( \frac{D_A}{2} \right)^2 \\
&= 0.291 \text{ Kg} \cdot \text{m}^2 + 1.87 \text{ Kg} \cdot \text{m}^2 = 2.16 \text{ Kg} \cdot \text{m}^2 = \text{I}_1
\end{align*}
\]

\[
\begin{align*}
\text{I}_2 : \quad \text{I}_2 &= \text{I}_{\text{body}} + 2 \cdot \text{I}_{\text{arms}} \\
&= 0.291 \text{ Kg} \cdot \text{m}^2 + 2 \cdot \left( \frac{1}{2} M_A \left( \frac{D_A}{2} \right)^2 + M_A \left( \frac{L_A}{2} + \frac{D_A}{2} \right)^2 \right) \\
&= 0.291 \text{ Kg} \cdot \text{m}^2 + 2.3 \text{ Kg} \left( \frac{0.05 \text{ m}}{2} + (0.05 \text{ m} + 0.115 \text{ m}) \right)^2 \\
&= 0.291 \text{ Kg} \cdot \text{m}^2 + 0.171 \text{ Kg} \cdot \text{m}^2 = 0.462 \text{ Kg} \cdot \text{m}^2 = \text{I}_2
\end{align*}
\]

\[\frac{\text{Solve for } \omega_2 \text{ in } \bigcirc}{\omega_2 = \frac{\text{I}_1 \cdot \omega_1}{\text{I}_2}} = \frac{1.87}{0.462} \cdot 8 \text{ rad/s} = \frac{1}{3} \text{ rad/s} \]

\[
\begin{align*}
\text{Total Angular Momentum is Constant} : \quad \text{I}_1 \cdot \omega_1 &= \text{I}_2 \cdot \omega_2 \\
\text{IX} \text{-AXIS THW.} \\
\end{align*}
\]

\[
\begin{align*}
\text{I}_1 &= \frac{1}{2} M_B \left( \frac{D_B}{2} \right)^2 + 2 \cdot \frac{1}{12} M_A \left( \frac{L_A}{2} \right)^2 + \frac{1}{4} M_A \left( \frac{D_A}{2} \right)^2 \\
&= \frac{1}{2} \cdot 44 \text{ Kg} \left( 0.115 \text{ m} \right)^2 + 2 \cdot \left[ \frac{(0.71 \text{ m})^2}{12} + \left( \frac{0.05 \text{ m}}{2} \right)^2 + \left( 0.355 \text{ m} + 0.115 \text{ m} \right)^2 \right] \text{ Kg} \\
&= 0.291 \text{ Kg} \cdot \text{m}^2 + 1.87 \text{ Kg} \cdot \text{m}^2 = 2.16 \text{ Kg} \cdot \text{m}^2 = \text{I}_1
\end{align*}
\]
Problem 4. (12 pts) You have a disk, with radius \( r = 30 \text{ cm} \) and uniformly distributed mass \( m_D = 2 \text{ kg} \), that can spin without friction about its symmetry axis (labeled as \( A \)). The disk is initially stationary. A particle with mass \( m_1 = 2 \text{ kg} \) is fixed to the edge of the disk. A second particle, with mass \( m_2 = 3 \text{ kg} \), travels at speed \( V = 0.25 \text{ m/s} \), at an angle \( \theta = 30^\circ \) from the \( y \)-axis, then strikes the first particle and sticks to it. How long after the collision (in seconds) will it take for the disk to complete one full revolution? Ignore gravity in this problem.

\[
\text{No external torques:}
\]

\[
\text{Use conservation of angular momentum around Point A. (2 pts)}
\]

- Initially: only angular momentum is in mass 2

\[
\begin{align*}
\text{A} & \cdot \vec{r} \\
\vec{V} & \rightarrow 30^\circ \\
\text{(Component \( \perp \) to radius \( r \))} & = V \cos 30^\circ = \frac{\sqrt{3}}{2} V
\end{align*}
\]

\[
\text{So } I_A = m_2 r^2 \text{ and } \vec{V} \cdot \vec{r} = \frac{(m_2 \cdot 0.25 \text{ m/s} \cdot 3 \text{ kg})}{2} = 0.195 \text{ kg} \cdot \text{m}^2
\]

- After collision: \( I_f = I_{\text{system}} \)

\[
I_{\text{system}} = I_{\text{disk}} + I_{\text{mass 1}} + I_{\text{mass 2}}
\]

\[
= \frac{1}{2} m_D r^2 + m_1 r^2 + m_2 r^2 = \left( \frac{1}{2} \cdot 2 \text{ kg} + 2 \text{ kg} + 3 \text{ kg} \right) r^2 = 6 \text{ kg} \cdot \text{m}^2
\]

- Momentum conservation \( |I_i| = |I_f| \Rightarrow 0.195 \text{ kg} \cdot \text{m}^2 = 0.54 \text{ kg} \cdot \text{m}^2 \)

\[
\text{So } \omega = 0.36 \frac{1}{s} = 0.36 \frac{\text{rad}}{s} \text{ (2 pts)}
\]

One revolution = \( 2\pi \text{ rad} \), so time for one revolution is

\[
T = \frac{2\pi}{0.36 \frac{\text{rad}}{s}} = \boxed{17.5 \text{ s}} \text{ (2 pts)}
\]

2 pts
Problem 5. (12 pts) Wile E. Coyote (*carnivorous vulgaris*) has devised a gadget for capturing Road Runner (*accelerati incredibus*) that consists of a cubic block (measuring 1 m on a side) with uniformly distributed mass \( m_2 = 200 \text{ kg} \), connected by a spring to a smaller block (0.3 m wide, 1 m tall and 1 m deep) that has uniformly distributed mass \( m_1 \). The blocks are initially touching, with the spring compressed; when Road Runner approaches the position shown in the figure, the coyote will activate the spring, which extends to a length \( L = 5 \text{ m} \). The blocks slide without friction on the ground. For what minimum value of \( m_1 \) will the cubic block crush the coyote flat against the wall on the right? The position \( x = 0 \) is the center of the cubic block at its initial position.

\[ m_1 \quad m_2 \]

\[ x = 0 \]

\[ -5 \text{ m} \quad 0 \quad 3 \text{ m} \]

\[ L \]

\[ 0.3 \text{ m} \quad 1 \text{ m} \]

\[ m_2 \]

\[ m_1 \]

- No external forces \( \rightarrow \) center of mass of device doesn't move.

**INITIAL:**

\[ x_{\text{com},i} = \frac{m_1 x_1}{M} = -\frac{m_1 (0.65 \text{ m})}{m_1 + m_2} \]

**FINAL:**

\[ x_{\text{com},f} = \frac{-m_1 (3.15 \text{ m}) + m_2 (2.5 \text{ m})}{m_1 + m_2} \]

\[ x_{1} = -0.65 \text{ m} \]

\[ x_{2} = 2.5 \text{ m} \]

\( (\frac{1}{2} (0.3+1) + L) \)

\[ x_{\text{com},i} = x_{\text{com},f} \quad \Rightarrow \quad -m_1 \cdot 0.65 = -m_1 \cdot 3.15 + m_2 \cdot 2.5 \]

\( m_1 \cdot 2.5 = m_2 \cdot 2.5 \)

\[ m_1 = m_2 = 200 \text{ kg} \]

(Could have anticipated this: since \( m_2 \)

has to move \( \frac{L}{2} \) to right, \( m_1 \) moves \( \frac{L}{2} \) to left.

5 For \( \text{com} \) to stay fixed, \( m_1 \) must = \( m_2 \).)