Problem 1. (10-60)

(a) The system is energy conserved.

For disk, though work-kinetic energy theorem
\[ \frac{1}{2} I w^2 = W_1 \]  
\[ \text{(1)} \]  
1 pt

For block
\[ \frac{1}{2} m v^2 = W_2 \]  
\[ \text{(2)} \]  
1 pt

For whole system
\[ W_1 + W_2 = mgh \]  
\[ \text{(potential energy)} \]  
\[ \text{(3)} \]  
1 pt

From (1)
\[ I = \frac{1}{2} m R^2 \]  
1 pt
Also
\[ v = wR \]

Put into (3)
\[ \frac{1}{2} I (wR)^2 + \frac{1}{2} m v^2 = mgh \]
\[ \Rightarrow \]  
\[ v^2 = \frac{4mgh}{M + 2m} \]  
2 pts

Put \( M = 400g \), \( m = 50g \), \( h = 50cm \), \( g = 9.8 \, \text{m/s}^2 \)
\[ \Rightarrow v = 1.4 \, \text{m/s} \]  
1 pt

(b) Since above calculation doesn't involve \( R \), therefore
\[ v = 1.4 \, \text{m/s} \]  
3 pts
Problem 2 (10-65)

The whole system is energy conserved.

Through the work-kinetic energy theorem

\[ \Delta k = W \quad \text{--- (1)} \]

Here \( W \) is the work done by the gravitational forces.

\[
W = W_{\text{hoop}} + W_{\text{rod}}
\]

\[ = mg h_1 + mg h_2 = mg(6R) + mg(2R) = 8mgR \quad \text{(1)} \]

(\( h_1 = 6R \), \( h_2 = 2R \) is from the distance of COM)

Kinetic energy \( \Delta k = \frac{1}{2} I W^2 - 0 = \frac{1}{2} I W^2 \quad \text{(2)} \)

\[ I = I_{\text{hoop}} + I_{\text{rod}} \]

\[ I_{\text{hoop}} = \frac{1}{2} mR^2 + m(2R)^2 \]

\[ I_{\text{rod}} = \frac{1}{12} m(2R)^2 + mR^2 \]

\[ \Rightarrow I = \frac{65}{6} mR^2 \quad \text{(3)} \]

Substitute (1) (2) (3) into (1)

\[ 8mgR = \frac{1}{2} \cdot \frac{65}{6} mR^2 W^2 \quad \Rightarrow W = \sqrt{\frac{96g}{65R}} \quad \text{(4)} \]

Take \( g = 9.8 \text{ m/s}^2 \), \( R = 0.15 \text{m} \)

\[ \Rightarrow W = 9.8 \text{ rad/s} \quad \text{(1 Pt)} \]
Problem 3  (11-4)

(a) Force diagram is sketched on the right

Gravitational force $\vec{F}_g$, frictional force $\vec{F}_f$

According to Second Newton's Law:

$$f_s = -mg\sin\theta = ma_{cm} x$$  \hspace{1cm} (1)  \hspace{1cm} \boxed{1\text{ Pt}}$$

Another equation can also be obtained after examining the ball:

$$Rf_s = I_{cm} a_{cm}$$  \hspace{1cm} (2)  \hspace{1cm} \boxed{1\text{ Pt}}$$

Also we have $a_{cm} = -2\alpha$  \hspace{1cm} (3)

Then

$$f_s = -I_{cm} \frac{a_{cm}}{R^2}$$  \hspace{1cm} (4)  \hspace{1cm} \boxed{1\text{ Pt}}$$

(Sign is inverted because $\alpha$ is positive, counter-clockwise)

Put (4) back into (1)

$$a_{cm} = -\frac{gs\sin\theta}{1 + \frac{I_{cm}}{MR^2}}$$  \hspace{1cm} \boxed{1\text{ Pt}}$$

As for balls, $I_{cm} = \frac{2}{5} MR^2$

$$\Rightarrow a_{cm} = -\frac{gs\sin\theta}{1 + \frac{2}{5}} = -0.1g$$  \hspace{1cm} \boxed{1\text{ Pt}}$$

$$\Rightarrow \sin\theta = 0.14 \Rightarrow \theta = 8.05^\circ \text{ or } 0.14 \text{ radian}$$
(b)
If a frictionless block slides down, Newton's second law says
\[-F_{gsin\theta} = ma_{\text{com}}\]
\[-mg \sin \theta = ma_{\text{com}}\]
\[\Rightarrow a_{\text{com}} = -g \sin \theta\]

Compared with Part (a)
\[\text{Part (a)} a_{\text{com}} = -\frac{F_{gsin\theta}}{1 + \frac{2}{5}}\]

\[|a_{\text{com}}^b| > |a_{\text{com}}|\]

\[\Rightarrow \text{At the same } \theta, \text{ acceleration magnitude of the block is more than } 0.1 g.\]
**Problem 4**  TEXT, 11-8.

1. \( M = 0.280 \text{g} \)  \( r \ll R \)  \( (R = \text{ball radius}) \)

\[
\begin{align*}
\text{Initial level:} & \quad h = 14.0 \text{ cm} \\
\text{Maximum height:} & \quad q \\
\end{align*}
\]

| What's \( h \) if the ball is on the verge of leaving the track at the top of the loop?
|-------------------|-------------------|
| \( \text{Consider forces there:} \) & \( \downarrow M\gamma \)  For ball to just stay on track, gravitational force is the centripetal force.
| \( q = \frac{V^2}{R} \Rightarrow V = \sqrt{2gR} \) & \( 1 \text{ pt} \)

To find \( h \): Use energy balance.

\[
\begin{align*}
\text{Potential E at top of ramp} & = \text{Pot. E at top of loop} + \text{K.E at top of loop} \\
Mgh & = Mg2R + \frac{1}{2} M \omega^2 + \frac{1}{2} MV^2 \quad \text{(2 pts)} \\
& = \frac{2}{5} M R^2 \quad \text{(Tab. 10-2)} \\
\omega & = \frac{V}{R} \quad \text{(Rounding)}
\end{align*}
\]

Rearrange

\[
\begin{align*}
Mg(h-2R) & = \frac{1}{2} \left( \frac{2}{5} M R^2 \right) \frac{V^2}{R^2} + \frac{1}{2} MV^2 = \left( \frac{1}{5} + \frac{1}{2} \right) MV^2 \\
& = \frac{7}{10} MV^2 = \frac{7}{10} \text{MgR} \quad \text{(using 2)} \quad \text{(2 pts)}
\end{align*}
\]

Solve for \( h \):

\[
\begin{align*}
h & = 2R + \frac{7}{10} R = \frac{27}{10} R = \frac{27}{10} (14 \text{ cm}) = 37.8 \text{ cm} = h
\end{align*}
\]

2 and 3. If \( h = 6.00R \), what's the magnitude and direction of the horizontal force component acting on the ball at point Q?^2

(Over)
Problem 4 - p. 2

Loop provides centripetal force: \( F_L = \frac{Mv^2}{R} \)

To find \( v \), use energy balance as in Part [a] (actually can solve directly for \( \frac{Mv^2}{R} \))

\[
Mgh = MgR + \frac{1}{10} MV^2 \\
\text{or} \quad \text{Potential Energy} \quad \text{Q} \\
\text{Rotational + Translational K.E.}
\]

Rearrange \( 5MgR = \frac{1}{10} MV^2 \)

\[
\text{So} \quad \frac{MV^2}{R} = F_L = \frac{50Mg}{7} = \frac{50 \cdot 0.28 \cdot 9.8}{5} = 0.020N = F_L
\]

Points to left. (2pts)

(1pt)
Problem 5 (11-14)

The total energy of current system is conserved.

At beginning, position P

\[ k_0 = \frac{1}{2} m v_0^2 + \frac{1}{2} I w_0^2 \]  
\[ I = \frac{2}{5} m r^2 \]

\[ \Rightarrow k_0 = \frac{1}{2} m v_0^2 + \frac{1}{10} m w_0^2 + \frac{7}{10} m v_0^2 = \frac{7}{10} m v_0^2 \]

1 pt

At position P'

\[ k_1 = \frac{1}{2} m v_1^2 + \frac{1}{2} I w_1^2 + m g h_1 + \frac{7}{10} m v_0^2 = \frac{7}{10} m v_1^2 + m g h_1 \]

2 pts

Then we are going to solve \( v_1 \)

Along x axis : \( d = v_1 t \)

Along y axis : \( h_2 = \frac{1}{2} g t^2 \)

\[ \Rightarrow v_1 = \frac{d}{t} = \frac{d}{\sqrt{2h_2/g}} \]

2 pts

Put \( \Theta \) into \( \Theta', \) and \( k_0 = k_1 \)

\[ \frac{7}{10} m v_0^2 = \frac{7}{10} m \frac{d^2 g}{2h_2} + m g h_1 \]

1 pt
\[ V_0^2 = \frac{c^2d}{2h_2} + \frac{10}{7} gh_1 \]  \( \text{Eq. (3)} \)

Put  
\[ g = 9.8 \text{ m/s}^2 \quad d = 0.06 \text{ m} \]
\[ h_1 = 0.05 \text{ m} \quad h_2 = 0.016 \text{ m} \]  \( \text{Substitute Eq. (3)} \)

\[ V_0 = 1.34 \text{ m/s} \]  \( 1 \text{ pt} \)
Problem 6. \((11-24)\)

According to the definition of Torque
\[ \vec{\tau} = \vec{r} \times \vec{F} \] \hspace{1cm} \text{(2 Pts)}

Here in the \(xyz\) coordinate system
\[ \vec{r} = (2 \text{ m}) \hat{i} - (3 \text{ m}) \hat{j} + (2 \text{ m}) \hat{k} \]
\[ \vec{F} = F_x \hat{i} + (7 \text{ N}) \hat{j} - (6 \text{ N}) \hat{k} \]

Recall the definition of the cross product.
\[ \vec{a} \times \vec{b} = (a_y b_z - a_z b_y) \hat{i} + (a_z b_x - a_x b_z) \hat{j} + (a_x b_y - a_y b_x) \hat{k} \] \hspace{1cm} \text{(2 Pts)}

Here
\[ \vec{r} \times \vec{F} = \text{(1 Pt)} \]
\[ = (18 - 14) \text{ N.m} \hat{i} + (12 + 2F_x) \text{ N.m} \hat{j} + (14 + 3F_x) \text{ N.m} \hat{k} \] \hspace{1cm} \text{(2 Pts)}

As \( \vec{\tau} = 4 \text{ N.m} \hat{i} + 2 \text{ N.m} \hat{j} - 1 \text{ N.m} \hat{k} \)

\[ \Rightarrow 12 + 2F_x = 2 \] \hspace{1cm} \Rightarrow \text{F}_x = -5 \text{ N} \hspace{1cm} \text{(1 Pt)}

\[ 14 + 3F_x = -1 \] \hspace{1cm} \Rightarrow \text{F}_x = -5 \text{ N} \hspace{1cm} \text{(verified)}