**Problem 1** (Text, Question 12.3)

\[ M_1 = 4.8 \text{ kg}. \]

Crossbars have negligible mass:

Stay horizontal.

What are \( M_2, M_3, M_4 \)?

* Need 3 Eqs. Consider torque balance for each crossbar.

* Top bar:

So \[ M_2 g L - (M_3 + M_4) g 3L = 0 \]

\[ \Rightarrow \frac{M_3 + M_4}{3} = \frac{M_2}{3} \]  

* Middle bar:

So \[ M_3 g L - M_4 g 3L = 0 \]

\[ \Rightarrow \frac{M_3}{3} = \frac{M_4}{3} \]

* Bottom bar:

\[ \Rightarrow \frac{M_2}{4} = \frac{M_1}{3} \]

To solve: Plug \( \star \star \) into \( \star \star \), get \[ \frac{4 M_3}{3} = \frac{M_2}{3} \] \[ \Rightarrow 4 M_3 = M_2 \]

\[ \Rightarrow 4 M_2 = M_1 \]

So \[ M_2 = \frac{M_1}{4} = 12 \text{ kg} \]

Then \( \star \star \star \star \) gives \[ M_3 = 3 \text{ kg} \]

And \( \star \star \star \star \) gives \[ M_4 = 1 \text{ kg} \]
Problem 2  (12-12)

Given  \( M_g = 40 \text{N} \)
\( m_g = 50 \text{N}, \phi = 35^\circ \)
\( T_2 \) horizontal

Find:  \( T_1, T_2, T_3, \theta \)

For the whole system in equilibrium
\[ T_1 \sin \phi = T_3 \sin \theta \]  \( x \) direction  --- (1)  
\[ T_1 \cos \phi + T_3 \cos \theta = 40 + 50 = 90 \]  \( y \) direction  --- (2)

For subsystem shown on the right, it is also in equilibrium
\[ T_1 \cos \phi = 40 \text{ along } y \]  --- (3)
\[ T_1 \sin \phi = T_2 \text{ along } x \]  --- (4)

For another subsystem shown on the right
Along \( y \) :  \( T_3 \cos \theta = 50 \)  --- (5)
Along \( x \) :  \( T_2 = T_3 \sin \theta \)  --- (6)

From (3)  \[ T_1 = \frac{40}{\cos \phi} = 48.8 \text{N} \]
From (4)  \[ T_2 = 40 \tan \phi = 28 \text{N} \]
From (5,6)  \[ \theta = \arctan \frac{T_2}{50} = 29.2^\circ \]
\[ T_3 = 57.3 \text{N} \]
Problem 3 (12-24)

Since the system is in equilibrium,
\[
\begin{align*}
\vec{F} &= 0 \\
\vec{C} &= 0
\end{align*}
\]
1 PT

Also, since vertical wall has no friction, only normal force to the wall, \( F_1 \), exists.

\[
\begin{align*}
F_x &= 0 \Rightarrow F_1 = f \\
F_y &= 0 \Rightarrow mg = F_2
\end{align*}
\]
1 PT

Torque at Point A

\[-F_1 \cdot \sqrt{L^2 - a^2} + mg \cdot d \cos \theta = 0\]

\[
\Rightarrow F_1 = \frac{mg \cdot d \cos \theta}{\sqrt{L^2 - a^2}}
\]
3 PTs

According to the definition of static friction coefficient

\[
\mu = \frac{\frac{f}{F_2}}{mg} = \frac{F_1}{mg} = \frac{d \cos \theta}{\sqrt{L^2 - a^2}} = \frac{d a}{L \sqrt{L^2 - a^2}}
\]
2 PTs

\( \mu = 0.2164 \)
Problem 4  (12-40)
The whole system is in equilibrium. Assume when fire fighter is in position $x$ the ladder is on verge of sliding.

Along $x$: $F_1 = f$  \[\text{1 PT}\]

Along $y$: $F_2 = Mg + mg$  \[\text{1 PT}\]

Torque about point $A$

$-F_1 \cdot h + mg \frac{1}{3} \cos \theta + Mg \cdot x \cos \theta = 0$  \[\text{2 PTS}\]

Since we know $\mu_s = 0.53 = \frac{f}{F_2}$  \[\text{1 PT}\]

$\Rightarrow f = 0.53 F_2 = 0.53 (Mg + mg) = F_1$  \[\text{4}\]

Put 4 into 3

$x = \frac{0.53(M + mh)}{M \cos \theta} - \frac{1}{3} \frac{m}{M}$  \[\text{2 PTS}\]

$m = 45 \text{ kg}, \ M = 72 \text{ kg}, \ l = 12 \text{ m}, \ h = 9.3 \text{ m}, \ \cos \theta = 0.632$  \[\text{1 PT}\]

$x = 10.1735 \text{ m}$, In percent: $\frac{x - \frac{l}{2}}{l} = 34.78\%$

Fire fighter should go 34.78% further.
Problem 5

\[ \overrightarrow{F_{AB}} = G \frac{m_{A}m_{B} \overrightarrow{d_{AB}}}{d_{AB}^2} \]

\[ = 6.67 \times 10^{-11} \frac{6 \times 10^{-3} \times 12 \times 10^{-3}}{0.5^2} \]

\[ = 1.921 \times 10^{-14} \, N \quad \text{along AB direction} \]

\[ F_{\text{net}} = 2.77 \times 10^{-14} \, N \quad \text{along -163.8°} \]

\[ \overrightarrow{F_{\text{net}}} = \overrightarrow{F_{AB}} + \overrightarrow{F_{Ac}} \]

\[ \Rightarrow \overrightarrow{F_{Ac}} = \overrightarrow{F_{\text{net}}} - \overrightarrow{F_{AB}} \]

To do this vector subtraction, there are two ways. You can do it graphically, or you can decompose into the coordinate system. Here, we use the second method.

Set up new coordinate system along \( F_{\text{net}} \) direction and perpendicular to \( F_{\text{net}} \) direction.

Along \( F_{\text{net}} \) direction:

\[ F_{AB} \cos(30° + 16.2°) + F_{Ac} \cos \phi = F_{\text{net}} \quad \text{(1)} \]

At right angles to \( F_{\text{net}} \) direction:

\[ F_{AB} \sin(46.2°) = F_{Ac} \sin \phi \quad \text{(2)} \]

From (1) and (2)

\[ \tan \phi = \frac{1.921 \sin 46.2°}{2.77 - 1.921 \cos 46.2°} = 0.9627 \Rightarrow \phi = 43.9° \quad \text{(1pt)} \]

\[ F_{Ac} = 2 \times 10^{-14} \, N \quad \text{(1pt)} \]
\[ F_{AC} = G \frac{M_A M_C}{d_{AC}^2} \]

\[ \Rightarrow d_{AC} = \sqrt{\frac{G M_A M_C}{F_{AC}}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^3 \times 8 \times 10^3}{2 \times 10^{-14}}} = 0.4 \text{ m} \]

The direction of \( F_{AC} \) is

\[-163.8^\circ + 43.9^\circ = -119.9^\circ \approx -120^\circ \]

from positive \( x \).

Therefore

\[ x_{\text{coordinate}} = -0.4 \cdot \sin 30^\circ = -0.2 \text{ m} \]

\[ y_{\text{coordinate}} = -0.4 \cdot \cos 30^\circ = -0.2 \sqrt{3} \approx -0.3464 \text{ m} \]
Problem 6 (13-28)

According to the definition of gravitational acceleration $\ddot{a}_g$

$$\ddot{a}_g = \frac{GM}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \cdot 1.98 \times 10^{30}}{(12 \times 10^3)^2}$$

$$= 9.17 \times 10^{11} \text{ m/s}^2$$

For free fall acceleration $g$

$F_{\text{n}} - mg = m(-w^2r)$

$$\Rightarrow mg = ma_g - m(w^2r)$$

$$\Rightarrow g = a_g - w^2r$$

Here $T = 0.041 \text{ s}, \quad T = \frac{2\pi}{w} \Rightarrow w = \frac{2\pi}{T}$

$$\Rightarrow g = a_g - \frac{4\pi^2}{T^2}r$$

Difference $|g - a_g| = \frac{4\pi^2}{T^2}r$ 

Percentage $= \frac{|g - a_g|}{a_g} = \frac{4 \times 3.14^2 \cdot 12 \times 10^3}{9.17 \times 10^{11}} = 3.07 \times 10^{-4}$