
Find the height of the apple when \( F = \frac{1}{2} F_e \) at the surface:

(a) outside the planet

(b) inside the planet

\[ F_{\text{apple-planet}} = \frac{G \text{apple} \cdot \text{M}_{\text{planet}}}{d^2} = \frac{1}{2} F_e = \frac{1}{2} \frac{G \text{apple} \cdot \text{M}_{\text{planet}}}{r_{\text{planet}}^2} \]

\[ \frac{G \text{M}_{\text{apple}} \cdot \text{M}_{\text{planet}}}{d^2} = \frac{1}{2} \frac{G \text{M}_{\text{apple}} \cdot \text{M}_{\text{planet}}}{r_{\text{planet}}^2} \]

Understanding of \( r \) in Eqs vs altitude

\[ d^2 = 2r_{\text{p}}^2 \]

\[ d = \sqrt{2} r_{\text{p}} \]

\[ \Rightarrow d = h + r_{\text{p}} \]

\[ h = d - r_{\text{p}} = \sqrt{2} r_{\text{p}} - r_{\text{p}} = 0.414 r_{\text{p}} \]

(b) Inside the planet, recall the Shell Theorem

\( \Rightarrow \) a shell exerts no net gravitational force on a particle inside of it.

\[ M_{\text{planet}} = \text{mass of planet which still creates a net gravitational force on the apple} \]

\[ F_{\text{apple-planet}} = \frac{G \text{M}_{\text{apple}} \cdot \text{M}_{\text{planet}}'}{d^2} = \frac{1}{2} F_e = \frac{1}{2} \frac{G \text{M}_{\text{apple}} \cdot \text{M}_{\text{planet}}'}{r_{\text{planet}}^2} \]

Assume uniform density:

\[ M_{\text{planet}}' = \rho \frac{4}{3} \pi d^3 \]

\[ M_{\text{planet}} = \rho \frac{4}{3} \pi r_{\text{p}}^3 \]

\[ M_{\text{planet}}' = \text{M}_{\text{planet}} \cdot \frac{d^3}{r_{\text{p}}^3} \]

\[ \frac{G \text{M}_{\text{apple}} \cdot \text{M}_{\text{planet}}'}{d^2} = \frac{1}{2} \frac{G \text{M}_{\text{apple}} \cdot \text{M}_{\text{planet}}'}{r_{\text{planet}}^2} \]

\[ \frac{d}{r_{\text{p}}^2} = \frac{1}{2} \frac{d}{r_{\text{p}}^2} \]

\[ d = \frac{r_{\text{p}}}{2} \Rightarrow h = r_{\text{p}} - d = r_{\text{p}} - \frac{r_{\text{p}}}{2} = \frac{1}{2} r_{\text{p}} \]
Problem 13-42

Given: \( m_B = m_C = m \)
\( D = 0.3057 \text{ m} \)
\( r_{AB} = r_{AC} = R \) (symmetry) as \( y \to 0 \) \( U \to -2.7 \times 10^{-11} \text{ J} \)

Find (a) \( m_B = m_C = m \)
(b) \( m_A \)

Potential Energy:
\[
U = -\frac{G m_B m_C}{R} \quad \text{(I)}
\]

\[
U_{\text{system}} = U_{AB} + U_{BC} + U_{AC}
= -\frac{G m_B m_A}{r_{AB}} - \frac{G m_C m_C}{r_{BC}} - \frac{G m_A m_C}{r_{AC}}
\]

But \( m_B = m_C = M \)
\( r_{AB} = r_{AC} = R \)
\( r_{BC} = 2D \)
\( r_{AC} = 2D \)

\[
= -\frac{G m_A M}{R} - \frac{G M M}{2D} - \frac{G m_A M}{2D} = -\frac{2G m_A M}{R} - \frac{G M^2}{2D} \quad \text{(II)}
\]

(a) \( \lim_{y \to 0} U_{\text{system}} = -2.7 \times 10^{-11} \text{ J} \) \( \Box \)

\[
\lim_{y \to 0} \left[ -\frac{2G m_A M}{R} - \frac{G M^2}{2D} \right] = \lim_{y \to 0} \left[ -\frac{2G m_A M}{\sqrt{D^2 + y^2}} - \frac{G M^2}{2D} \right]
= -\frac{G M^2}{2D} = -2.7 \times 10^{-11}
\]

\[
M^2 = \frac{2D}{G} (2.7 \times 10^{-11}) = 2 \left( 0.3057 \text{ m} \right) \frac{(2.7 \times 10^{-11})}{\text{kg}^2} \text{m}^2 \text{kg}^{-2}
\]

\( M = 0.497 \text{ kg} \) = mass of particles B & C \( \Box \)

(b) \( \lim_{y \to 0} U_{\text{system}} = -3.5 \times 10^{-11} \text{ J} \) (from plot) \( \Box \)

\[
\lim_{y \to 0} \left[ -\frac{2G m_A M}{R} - \frac{G M^2}{2D} \right] = \lim_{y \to 0} \left[ -\frac{2G m_A M}{\sqrt{D^2 + y^2}} - \frac{G M^2}{2D} \right]
= -2G m_A M - \frac{G M^2}{2D} = -3.5 \times 10^{-11}
\]

\[
m_A = \left[ -3.5 \times 10^{-11} + \frac{G M^2 - D}{2D} \right] \frac{1}{2GM}
\]
(cont.)

\[ M_A = \left[ -3.5 \times 10^5 + \frac{6.67 \times 10^{-11} \text{ m}^3}{\text{kg}^2} \left( 0.497 \text{ kg} \right)^2 \right] - \frac{0.3057}{2(6.67 \times 10^{-11})(0.497\text{ kg})} \]
\[ = 1.4896 \text{ kg}, \ 	ext{mass of particle A} \]

**Problem 3** problem 13-52

Given: A Satellite in geosynchronous orbit over the equator
Find: the attitude

To stay in orbit over the same spot on earth, the period of the satellite must be the same as earth: 1 day

\[ T = 1 \text{ day} \]
\[ = \frac{2\pi}{1 \text{ day}} \frac{\text{day}}{24 \text{ hours}} \frac{\text{hour}}{60 \text{ min}} \frac{\text{min}}{60 \text{ sec}} = 0.000073 \text{ rad/sec} \]

For the satellite not to fall, the centripetal force must be balanced by the gravitational force

\[ F_c = F_g \]

\[ \frac{G M M}{r^2} \]

\[ r \left( \frac{2\pi}{T} \right)^2 = \frac{G M M}{r^2} \]

\[ r^3 = \frac{G M M}{\left( \frac{2\pi}{T} \right)^2} \Rightarrow r = \frac{3}{\left( \frac{2\pi}{T} \right)^2} \]

\[ = \frac{3}{\left( \frac{2\pi}{0.000073\text{ rad/sec}} \right)^2} \]

\[ = 4.225 \times 10^7 \text{ m} \]

Altitude is distance above Earth

\[ h = r - r_e \]

\[ = (4.225 \times 10^7 - 6.37 \times 10^6) \text{ m} \]

\[ = 3.588 \times 10^7 \text{ m} \]
An asteroid, Ida, is orbited by a small moon, Dactyl.

$a_{moon} = 1.5 \text{ km} \\
T = 100 \text{ km} \\
D_{moon} = 55 \text{ km}

Assume circular orbit with $T = 27 \text{ h}$

Find:
(a) Mass of asteroid
(b) If $V_{asteroid} = 14100 \text{ km}^3$
    find $\rho$

For the moon to stay in orbit:

(a) \[
F_c = F_g \\
F_c = \frac{G m_{moon} M_{asteroid}}{r^2} \\
F_c = \frac{G m_{moon} m_{asteroid}}{r^2} \\
w^2 r = \frac{G M_{asteroid}}{r^2} \\
M_{asteroid} = \frac{w^2 r^3}{G} \\
= \left( \frac{2\pi}{27 \text{ h}} \right)^2 \left[ \frac{100 \text{ km}}{1000 \text{ km}} \right]^3 \\
= \frac{6.26 \text{e-11} \text{ m}^3}{\text{kg} \cdot \text{sec}^2} \\
= 6.26 \text{e}16 \text{ kg}
\]

(b) $\rho = \frac{M_{asteroid}}{V_{asteroid}} = \frac{6.26 \text{e16} \text{ kg}}{14100 \text{ km}^3 \cdot (1000 \text{ m}^3/\text{km})^3}$

$= 4443.1 \text{ kg/m}^3$
Problem 13-68

2 space ships start out in Circular Orbits around Earth. Igor is travelling 90° ahead of Picard.

$h = 400\text{ km}$ altitude, $M = 2000\text{ kg} = M_i = M_p$

Find (a) $T_i$ period
(b) $V_o$ Speed

Initially $T_i = T_p$ & Picard

must be true or else the com would change, and
this is not possible with no external forces.

$T_i = T_p = T_0$

To stay in orbit the centripetal force must balance the gravitational force.

\[ F_c = F_g \]
\[ \frac{M_e}{r^2} = \frac{GM_E}{r^2} \]

\[ \omega^2 r = \frac{GM_E}{r^2} \]
\[ \frac{V_o^2}{r^2} r = \frac{GM_E}{r^2} \]
\[ V_o^2 = \frac{GM_E}{r} \Rightarrow V_o = \sqrt{\frac{GM_E}{r}} \]

\[ r = h + r_{\text{Earth}} \]
\[ V_o = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{\left(6.67 \times 10^{-11} \text{ m}^3/\text{kg s}^2\right) 5.98 \times 10^5 \text{ kg}} {400 \text{ km} + 6.37 \times 10^6 \text{ m}}} \]
\[ = 7675.72 \text{ m/s} \]

(b) \[ T_0 = \frac{2\pi}{V_o} = \frac{2\pi r}{V_0} = \frac{2\pi r}{V_0} = \frac{2\pi \left(400 \text{ km}, 1000 \text{ km} + 6.37 \times 10^6 \text{ m}\right)}{7675.72 \text{ m/s}} \]

(a) \[ = 5541.78 \text{ s} = 1.539 \text{ hrs} \]
At point P, Picard fires a burst which reduces his speed by 1.00%.

Find(c) \( KE_f \)

(d) \( v_f \) (immediately after the burst)

(e) In the new orbit, find \( E \)

(f) \( \alpha \)

(g) \( T \)

(h) how much earlier will Picard arrive back at \( P \).

(c) \[
\begin{align*}
V_f &= V_0 - 0.01 \times V_0 \\
&= 0.99 \times V_0 \\
KE_f &= \frac{1}{2} m v_f^2 \\
&= \frac{1}{2} (2000 \text{ kg}) (0.99 \times 7675.72 \text{ m/s})^2 \\
&= 5.77 \times 10^9 \text{ J}
\end{align*}
\]

Note: Mass of Earth \( M_E = 5.98 \times 10^{24} \text{ kg} \)

radius of Earth \( r_E = 6.37 \times 10^6 \text{ m} \)

(Table 13.2)

(d) After the burst, Potential energy remains the same:

\[
U_f = \frac{G m M_E}{r} = \frac{6.67 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2 (2000 \text{ kg}) (5.98 \times 10^{24} \text{ kg})}{(900 \text{ km} / 1000 \text{ m/km} + 6.37 \times 10^6 \text{ m})} \\
= -1.17 \times 10^{11} \text{ J}
\]

(e) Total Energy

\[
E_f = U_f + KE_f = 5.77 \times 10^9 \text{ J} + -1.17 \times 10^{11} \text{ J} = -6.01 \times 10^9 \text{ J}
\]
(f) \( E = \frac{-GMm}{2a} = 6.01 \times 10^5 \text{ J} \) from part (e)

\[
\alpha = \left(\frac{6.67 \times 10^{-11} \text{ m}^3/\text{kg}^2 \cdot (5.98 \times 10^5 \text{ kg})}{2 \times (6.01 \times 10^5 \text{ J})}\right)^{\frac{1}{2}} \Rightarrow 6637.9 \text{ km}
\]

(g) Kepler's 3rd Law:

\[
T_f^2 = \left(\frac{4\pi^2}{GM}\right)\alpha^3
\]

\[
= \frac{4\pi^2}{(6.67 \times 10^{-11} \text{ m}^3/\text{kg}^2 \cdot (5.98 \times 10^5 \text{ kg}))^3}
\]

\[
= 2.894 \times 10^7 \text{ s}^2
\]

\[
T_f = \sqrt{2.894 \times 10^7 \text{ s}^2} = 5380.38 \text{ s}
\]

(h) What is the time difference?

Initially, Igor has a 90s head start. Call \( t = 0 \) when Igor passes P.

Igor returns at \( t_f = 70 \text{ s} \) secs

\[
t_p = 90 \text{ s} + T_f = 90 + 5380.38 \text{ s} = 5470.38 \text{ s}
\]

Picard takes 90s to get to P+P, then decelerates

How much earlier:

\[
\Delta t = |t_p - t_f| = |5470.38 \text{ s} - 5541.78 \text{ s}| = 71.4 \text{ seconds}
\]
Problem 6 (text problem 13-99)

Gravitational attraction on mass \( m \).

Note: Net force is on x-axis only — radial part cancels around the ring.

- Consider a small piece of the ring with mass \( dm \). The gravitational force on \( m \) from \( dm \) is:

\[
F_g = \frac{G \cdot m \cdot dm}{r^2} = \frac{G \cdot m \cdot dm}{x^2 + r^2} \quad \text{(acting on the line between \( m \) and \( dm \))}
\]

The x-component is:

\[
F_{gx} = \frac{F_g \cdot \cos \theta}{r} = \frac{G \cdot m \cdot x}{(x^2 + r^2)^{3/2}}
\]

- The total gravitational force is:

\[
\int_{(\text{entire ring})} \frac{G \cdot m \cdot x}{(x^2 + r^2)^{3/2}} \, dm = \frac{G \cdot m \cdot m}{(x^2 + R^2)^{3/2}} \quad \text{toward the center of the ring}
\]

6. Suppose the mass \( m \) starts from rest at \( x \). What's its velocity as it passes through the center of the ring?

\[m \overset{\text{kinetic energy}}{\rightarrow} x\overset{\text{potential energy}}{\rightarrow} 0\]  

(\text{final state}) \overset{\text{kinetic energy}}{\rightarrow} \overset{\text{potential energy}}{\rightarrow} \overset{\text{kinetic energy}}{\rightarrow} \text{(initial state)}

\[
\frac{1}{2} m v^2 = U(x) - U(x=0) = -\frac{G \cdot m \cdot M}{\sqrt{x^2 + R^2}} + \frac{G \cdot m \cdot M}{R}
\]

So \( v^2 = 2GM \left[ \frac{1}{R} - \frac{1}{\sqrt{x^2 + R^2}} \right] \)

And \( v = \sqrt{2GM \left[ \frac{1}{R} - \frac{1}{\sqrt{x^2 + R^2}} \right]} \)