Problem 1 - Short answers. (Total 10 pts)

(a) (5 pts) Consider an elevator cab that contains water (density $\rho = 1000 \text{ kg/m}^3$) to a depth of $h = 0.8 \text{ m}$. The cab has an open service door on top, so that the air in the cab is at atmospheric pressure, $p_{\text{ATM}}$. Suppose that (somehow) the cables holding the cab all fail simultaneously and the cab goes into free fall. What's the pressure in the water at the bottom of the tank? Explain.

\[ \frac{\partial p}{\partial z} = 0 \] (No variation in $z$)

\[ a_z = -g \]

So \( \frac{\partial p}{\partial z} = 0 \), no $p$ variation in $z$!

\[ p = p_{\text{ATM}} \] Throughout the cab (including at the bottom).

(b) (5 pts) An airplane is in level flight at constant speed $V$, in still air. At some point the airplane passes directly over a balloon that previously sat perfectly still in the atmosphere. After the plane passes, does the balloon move upward, downward, or stay at the same altitude? Explain briefly.

\[ V \]

\[ \text{UPWARD} \]

\[ \text{HAZORZNTAL: AIR ISSUES HERE} \]

Air imposes lift (and drag) forces on the plane

$\Rightarrow$ Plane imposes a downward reaction force on the air (forces balance)

Air gets deflected downward after plane passes,

Balloon gets blown downward.
Problem 2. (Total 20 pts) Consider two large fluid reservoirs, each open at the top and having a square cross-section spanning $W = 3\text{ m}$ on a side. The reservoirs are connected by a long channel, which has a square cross-section having side length $L = 0.5\text{ m}$. The reservoir on the left contains a liquid with density $\rho_1 = 800\text{ kg/m}^3$, while the liquid in the reservoir on the right has density $\rho_2 = 1000\text{ kg/m}^3$. The fluid on the left has depth $H_1 = 10\text{ m}$, and the fluid on the right has depth $H_2 = 7\text{ m}$, measured from the top of the channel. There is a door in the channel, hinged on the bottom, that separates the liquid in the two reservoirs.

![Diagram of two reservoirs connected by a channel with a door]

\[ \rho_1 = 800\text{ kg/m}^3 \quad \text{hinge} \quad \rho_2 = 1000\text{ kg/m}^3 \]

a) (18 pts) What torque (magnitude and direction) needs to be applied at the hinge to keep the door closed?

b) (2 pts) If the door were allowed to swing freely (i.e. if no torque were applied to the hinge) in which direction would liquid flow in the channel? Explain briefly.

\[ \text{a) Determine force line of action for each side separately} \]

\[ \text{LEFT SIDE} \]

\[ F_L = \rho \left( h_L = 10.25\text{ m} \right) \cdot A_{\text{door}} = \rho_1 g h_L \cdot A_{\text{door}} = \rho_1 g L^2 \left( = 2.009 \cdot 10^4 \text{ N} \right) \]

\[ \text{RIGHT SIDE} \]

\[ F_R = \rho \left( h_R = 7.25\text{ m} \right) \cdot A_{\text{door}} = \rho_2 g h_R \cdot L^2 \left( = 1.176 \cdot 10^4 \text{ N} \right) \]

\[ Y_L' = h_L + \frac{\rho g}{F_L} \cdot I_{XX} \]

\[ Y_R' = h_R + \frac{\rho_2 g}{F_R} \cdot I_{XX} \]

\[ Y_L' = h_L + \frac{L^2}{12h_L} \left( = 10.252 \text{ m} \right) \]

\[ Y_R' = h_R + \frac{L^2}{12h_R} \left( = 7.253 \text{ m} \right) \]
Door closed → no torque on hinge = 0

\[ T = F_L \left( \frac{H_1 + L}{L} - Y_L' \right) + F_R \left( \frac{H_2 + L}{L} - Y_R' \right) = 0 \]

So

\[ T = 2.009 \times 10^4 \text{N} \cdot \left( 10.5 \text{m} - 10.252 \text{m} \right) - 1.776 \times 10^4 \text{N} \cdot \left( 7.5 \text{m} - 7.253 \text{m} \right) \]

\[ T = 5.96 \times 10^2 \text{N} \cdot \text{m} \]

Counter clockwise → need to apply this to keep the door closed.

If no applied torque → force moments from liquids would push the door open clockwise, so liquid would flow from left to right.
**Problem 3.** (20 pts) A square sled, with length \( L = 2 \text{ m} \) (and width \( L \) in the out-of-page direction), rides on a layer of SAE 10W oil (dynamic viscosity \( \mu = 0.1 \text{ N·s/m}^2 \)) with thickness \( h \). The sled carries a tank of water (density \( \rho = 998 \text{ kg/m}^3 \)), as well as a water jet inclined at \( 15^\circ \) to the horizontal that shoots a stream of water with speed \( V_J = 20 \text{ m/s} \) (measured relative to the sled) and diameter \( d = 5 \text{ cm} \).

If the sled moves to the left at a constant speed \( V_S = 3 \text{ m/s} \), what's \( h \)? (Ignore the effect of gravity on the water jet, and assume that the water in the tank is stationary with respect to the sled. Also, all water moves parallel to the rigid wall shown after striking it.) Solve this using an appropriate control volume.

\[
\begin{align*}
\text{(C.V.} & \rightarrow \text{Surface between Sled/Oil} \rightarrow x \text{ (Note: The} \quad \text{Rigid Wall} \quad \text{is Irrelevant} \quad \text{to this problem)}) \\
\text{(2)} \quad (\text{Moves with Sled}) & \quad \text{Force on Sled from Oil} \\
\text{Can get appropriate } h \text{ from } F.
\end{align*}
\]

• Write momentum eq. (in \( x \)-direction)

\[
\sum F_{x,CV} = F = \frac{\partial}{\partial t} \int_0^L \rho V_x \text{d}A + \int_0^L \rho V_x (\text{d}V_x) \quad (\text{gives } F \text{ for case of no acceleration})
\]

**Term \( [A] \):** In tank, \( V_x \) (relative to C.V.) = 0

In jet, \( V_x \neq 0 \), but volume of fluid in jet in C.V. is fixed (as \( V_x \) is fixed) \( \Rightarrow \frac{\partial}{\partial t} (\ldots) = 0 \)

\[
\text{So } \Rightarrow [A] = 0 \quad (3)
\]
(Extra workspace for problem 3)

**Term 2**: Non-zero only where jet crosses control surface

\[ F \leq \rho \frac{V_j^2 \pi d^2 \cos 15^\circ}{4} = 0.18 \text{ kg/m}^3 \cdot (20 \text{ m/s})^2 \pi \left( \frac{0.05 \text{ m}}{4} \right)^2 \cdot 0.966 = 757 \text{ N} \]

\[ \dot{Q} = \rho \left( V_j \cos 15^\circ \right) \cdot \frac{\pi d^2}{4} \]

\[ \dot{Q} > 0: \text{ points in } +x \text{ direction} \]

\[ \dot{Q} > 0: \text{ outflow from C.V.} \]

\[ h = \frac{1}{F} \mu \frac{V_0 L^2}{h} = \frac{1}{757 \text{ N}} \cdot 0.1 \text{ kg/m}^2 \cdot 3 \text{ m} \cdot (2 \text{ m})^2 \]

\[ h = 1.59 \times 10^{-3} \text{ m} = 1.59 \text{ mm} \]
Bonus questions from the world of the arts.
(The top four scorers win a free lunch.)

1. Who wrote *War and Peace*?
   a) Ulysses S. Grant  
   b) Leo Tolstoy  
   c) Fyodor Dostoevsky  
   d) Boris Pasternak

2. Which of these composers wrote a fiftieth symphony?
   a) Beethoven  
   b) Mozart  
   c) Haydn  
   d) Tchaikovsky

3. Which of these artists is famous for his painting of the U.S. flag?
   a) Andy Warhol  
   b) Jackson Pollock  
   c) Jasper Johns  
   d) Robert Rauschenberg

4. This actor was a bomber pilot in World War II:
   a) Cary Grant  
   b) Gregory Peck  
   c) Sean Penn  
   d) Jimmy Stewart

5. Which of these *Beatles* songs came earliest?
   a) *Please Please Me*  
   b) *Revolution*  
   c) *Help!*  
   d) *Let It Be*