Problem 1. (10 pts) The control volume shown has three sides: side 1 is a half-circle with diameter \(d\), while sides 2 and 3 have identical length and meet with vertex angle \(\theta = 90^\circ\). The control volume extends a length \(l\) in the out-of-page direction. Suppose you have a steady, uniform fluid flow where all fluid particles move in the same direction, in the plane of the page, with velocity \(U\). The flow direction makes an angle of 30\(^\circ\) with the symmetry axis of the control volume. Compute the flux term in the mass conservation equation (Eq. 4.12 in the text) for faces 1, 2, and 3 separately by explicitly evaluating the appropriate integrals.

Problem 2. (15 pts) Since your uncle owns a very productive cucumber farm, you sense a business opportunity in pickles. Your business uses the tank arrangement shown below to make the necessary pickling brine. A salt solution (concentration 240 grams of salt per liter of water) flows out of a faucet at a constant volume flow rate \(Q = 1.2\) L/s and enters the tank. The tank contains 900 liters of water and has a regulated valve on the side that maintains a constant liquid depth, \(h\). Assume that the water in the tank is initially free of salt, that dissolving salt in water doesn’t change the volume of the water, that the salt in the tank is uniformly distributed at all times, and that everything is at a uniform temperature of 20\(^\circ\)C. How long will it take for the water in the tank to contain 130 grams of salt per liter of water? Solve this problem using the control volume equation, Eq. 4.10, and define your control volume carefully (there is one possible definition that we know that is far superior to the other). In applying this equation, define the density, \(\rho\), to be the density of the water only (ignoring the dissolved salt). With that definition of \(\rho\), explain briefly how you define \(N\), \(dN/dt\), and \(\eta\).
**Problem 3.** (10 pts) Consider the device shown, which takes in a jet with velocity $V_1 = 10 \text{ ft/s}$ in the $x$-direction and diameter $d_1 = 2 \text{ in}$, then diverts the flow into two jets, one that makes an angle of $70^\circ$ with the $x$-axis and has velocity $V_2 = 8 \text{ ft/s}$ and diameter $d_2 = 2.2 \text{ in}$, and one that makes an angle of $-30^\circ$ with the $x$-axis and has velocity $V_3 = 15 \text{ ft/s}$ and unknown diameter $d_3$. The water is all at $70^\circ \text{ F}$, the three jets lie in the horizontal plane, and the amount of water inside the device itself is constant. What’s $d_3$ in this case, and what force (magnitude and direction) is required to hold the device stationary?

**Problem 4.** (15 pts) Water flows out of a faucet, which has diameter $d = 3 \text{ cm}$, at a volume flow rate $Q = 0.5 \text{ liters/sec}$ and uniform velocity $v_e$. At a distance $h$ from the faucet exit, the water strikes a horizontal plate, after which all of the water flows horizontally along the surface of the plate. As we will confirm later, the water accelerates due to gravity as it flows downward away from the faucet, with the water velocity at the position $h$ being given by $v(h) = \sqrt{v_e^2 + 2gh}$, where $g$ is gravitational acceleration. What force, $F$, is necessary to hold the plate in place if $h = 2 \text{ cm}$? What’s $F$ if $h = 20 \text{ cm}$? Explain physically why these answers are different.