1. Continuity:

\[ V_1 \frac{A_1}{A_2} = V_2 \frac{A_2}{A_1} \]

\[ \Rightarrow V_1 \frac{\pi d_1^2}{4} = V_2 \frac{\pi d_2^2}{4} \]

\[ V_1 d_1^2 = V_2 d_2^2 \quad (1) \]

Bernoulli from \( \text{a} \) to \( \text{b} \)

\[ \frac{1}{2} p V_1^2 + p_i + p g h = \frac{1}{2} p V_2^2 + p_{\text{ atm}} \quad (\times) \]

unknown.

To get \( p_1 \), use the force balance for the piston

\[ p_{\text{ atm}} A_1 + mg = p_1 A_1 \]

\[ \Rightarrow p_1 = p_{\text{ atm}} + \frac{mg}{A_1} \quad (2) \]

plug eq (1), (2) into (\( \times \)), and

\[ \frac{1}{2} p \left[ V_2 \left( \frac{d_2}{d_1} \right)^2 \right] + \left[ p_{\text{ atm}} + \frac{mg}{A_1} \right] + p g h \]

\[ = \frac{1}{2} p V_2^2 + p_{\text{ atm}} \]

\[ \Rightarrow V_2 = \sqrt{2 \left( \frac{mg}{\frac{A_1}{A_2}} + gh \right)} \quad (3) \]

\[ = \sqrt{2 \left( \frac{0.4 \times 10}{0.8 \times 0.01 \times 0.01^2} + 10 \times 9.8 \right)} \]

\[ = 2 \text{ m/s} \]
\[ Q_{out} = V_2 A_2 = V_2 \frac{\pi d_2^2}{4} \]
\[ = 2 \times \frac{\pi}{4} \times (0.5 \times 10^{-3})^2 \]
\[ = \boxed{3.9 \times 10^{-7} \text{ m}^3/\text{s} \quad \text{or} \quad 0.39 \text{ cm}^3/\text{s}} \]

(1)

To double the flow rate is to double the outlet velocity \( V_2 \), since \( A_2 = \text{const.} \).

From (3), we know

\[ V_2 \propto \sqrt{\frac{m}{\rho A_1 + h}} \]

or \( V_2^2 \propto \left( \frac{m}{\rho A_1 + h} \right) \)

i.e. \( \left( \frac{V_2'}{V_2} \right)^2 = \frac{\frac{m'}{\rho A_1 + h}}{\frac{m}{\rho A_1 + h}} \)

(2)

For \( \frac{V_2'}{V_2} = 2 \),

\[ \frac{m'}{\rho A_1 + h} = 4 \left( \frac{m}{\rho A_1 + h} \right) \]

\( \Rightarrow \)

\[ m' = 4m + 3 \rho A_1 h \]

\[ = 4 \times 0.4 + 3 \times 10^3 \times \left( \frac{\pi}{4} \times 0.1^2 \right) \times 0.15 \]

\[ = \boxed{5.1 \text{ kg}} \]

(1)
2.

1. Flow vel. at the exit

Bernoulli from the water surface to the exit:

\[
\frac{1}{2} \rho V_{in}^2 + P_{atm} = \frac{1}{2} \rho V_{ex}^2 + P_{atm} - \rho gh \quad (\gamma)
\]

Since the tube is sufficiently large

\[\Rightarrow V_{ex} = \sqrt{2gh} \quad (\beta)\]

For the left config. \[V_{ex} = \sqrt{2gL}\]

For the right config. \[V_{ex} = \sqrt{2g \cdot 2L} = 2 \sqrt{gL}\]

Plug in numbers.

Left: \[V_{ex} = \sqrt{2 \times 10 \times 0.2} = 2 \text{ m/s} \quad (\alpha)\]

Right: \[V_{ex} = 2 \sqrt{10 \times 0.2} = 2.8 \text{ m/s} \quad (\alpha)\]

2. Pressure at the apex.

Bernoulli from water surface to the apex.

\[P_{atm} = \frac{1}{2} \rho V_{c}^2 + P_c + \rho gh \quad (\gamma)\]

From continuity, since the area is the same everywhere in the tube, so as the velocity, therefore.

\[V_c = V_{ex}\]

\[P_c = P_{atm} - \frac{1}{2} \rho V_{ex}^2 - \rho gh \quad (\alpha)\]
For the left config:

\[ P_c = P_{atm} - \frac{1}{2} \rho \left( \sqrt{2gL} \right)^2 - \rho g H \]

\[ = P_{atm} - \rho g (L + H) \]

\[ = 10^5 - 10^3 \times 10 \times (0.2 + 0.1) \]

\[ = 0.97 \times 10^5 \text{ Pa} \quad \text{(1)} \]

For the right one:

\[ P_c = P_{atm} - \frac{1}{2} \rho \left( 2\sqrt{gL} \right)^2 - \rho g H \]

\[ = P_{atm} - \rho g (2L + H) \]

\[ = 10^5 - 10^3 \times 10 \times (2 \times 0.2 + 0.1) \]

\[ = 0.95 \times 10^5 \text{ Pa} \quad \text{(1)} \]

(3). Pressure at \( L \) below the free surface.

For the left config, this position is open to the atmosphere. So \[ P_{exit} = P_{atm} \quad \text{(1)} \]

For the right config, use Bernoulli from \( B \) to the exit:

\[ \frac{1}{2} \rho V_B^2 + P_B + \rho g L = \frac{1}{2} \rho V_{ex}^2 + P_{atm} \quad \text{(1)} \]

\[ V_B = V_{ex} \text{ from continuity} \]

\[ \Rightarrow \quad P_B = P_{atm} - \rho g L \]

\[ = 10^5 - 10^3 \times 10 \times 0.2 \]

\[ = 0.98 \times 10^5 \text{ Pa} \quad \text{(1)} \]
(4). Pressure at 2 L below the free surface.

For the left config., this position is open to the atmosphere, so the pressure \( P_a = P_{atm} \). ①

For the right config., this position is open to the atmosphere, \( P_{exit} = P_{atm} \). ②

(5). Cavitation means that the pressure is below vapour pressure, which can be assumed to be zero in comparison with the \( P_{atm} \).

Thus \( P_c \) must be positive non-negative.

For the left config.

\[ P_c = P_{atm} - P_g (L + H) \geq 0 \] ①

\[ \Rightarrow H \leq \frac{P_{atm}}{P_g} - L \]

\[ = \frac{1.05}{10} - 0.2 = 0.8 \ m \] ①